DIGITAL IMAGE PROCESSING AND ENHANCEMENT
Lecture 17 — Session 1, 2009
Bispectrum in Speckle Restoration

DIP&E-17.1 Bispectrum in Speckle Restoration

The bispectrum provides a powerful approach to speckle phase restoration.

- *The bispectrum, which is four-dimensional, can be computed from the 2D Fourier transform of an image (discussed next).*

A sequence of image frames of a stationary object is captured in the usual way by telescope.

- *Each frame will be distorted and blurred by a random speckle PSF, due to changing atmospheric turbulence conditions.*
- *The bispectrum is computed of each frame, and averaged over the sequence.*

In the bispectrum, random phase errors due to turbulence cancel out on average.

The final step is to compute the corrected 2D Fourier phases from the bispectrum.

DIP&E-17.2 Bispectrum in Speckle Restoration

What is the bispectrum, and how is it computed?

- *The bispectrum, $I^{(3)}(u_1,u_2)$, is the Fourier transform of the triple correlation, $i^{(3)}(x_1, x_2)$. These are defined as follows:*

$$i^{(3)}(x_1,x_2) = \int i^*(x)\, i(x+x_1)\, i(x+x_2)dx \,,$$

$$I^{(3)}(u_1,u_2) = I(u_1)\, I(u_2)\, I^*(u_1+u_2) \,, \qquad (1)$$

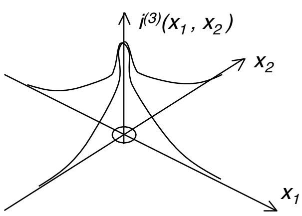
where i(x) represents the signal and I(u) its Fourier transform.

Note that this definition is for a 1D signal, i(x) , and results in a 2D triple correlation and a 2D bispectrum.

- *If the signal is a 2D image, then its triple correlation is 4D, and its bispectrum is also 4D.*
- *Thus, the amount of data produced may be very large (e.g., N^4).*

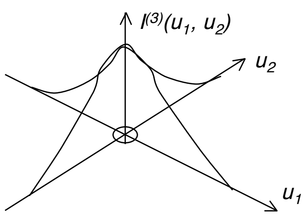
DIP&E-17.3 Bispectrum in Speckle Restoration

Diagrammatically, we can visualise a triple correlation and bispectrum as follows:



Triple correlation of 1D signal i(x)

Note: x_1 and x_2 are independent delays over which correlation found.



Bispectrum of 1D signal i(x)

Note: u_1 and u_2 are spatial frequencies selected for combination in Eq. (1).

DIP&E-17.4 Bispectrum in Speckle Restoration

Reconstruction of phase of the signal Fourier transform, I(u), from the bispectrum can occur as follows:

- *The phase of the bispectrum is given by:*

  $$\beta^{(3)}(u_1,u_2) = \beta(u_1) + \beta(u_2) - \beta(u_1 + u_2) \ ,$$

  where $\beta^{(3)}(.)$ *is the phase of the bispectrum and* $\beta(.)$ *is the phase of the signal Fourier transform.*
- *For a 1D signal, if we choose* $u_2 = 1$*, then:*

  for $u_1 = 1 \quad \beta(2) = \beta(1) + \beta(1) - \beta^{(3)}(1,1)$,

  for $u_1 = 2 \quad \beta(3) = \beta(1) + \beta(2) - \beta^{(3)}(2,1)$,

  for $u_1 = 3 \quad \beta(4) = \beta(1) + \beta(3) - \beta^{(3)}(3,1)$,

  $\vdots$

  for $u_1 = n \quad \beta(2) = \beta(1) + \beta(n-1) - \beta^{(3)}(n-1,1)$.

DIP&E-17.5 Bispectrum in Speckle Restoration

But, the phase of the Fourier coefficient at index 1 simply determines the position of the object, and can be chosen arbitrarily (if position can be adjusted later).

- *So, arbitrarily setting* $\beta(1) = 0$ *allows computation of* $\beta(2)$ *by:*

  $$\beta(2) = -\beta^{(3)}(1,1) \, .$$

- *Similarly, all the other values can be computed:*

  $\beta(3) = \beta(2) - \beta^{(3)}(2,1)$,

  and $\beta(4) = \beta(3) - \beta^{(3)}(3,1)$,

  $\vdots$

  and $\beta(n) = \beta(n-1) - \beta^{(3)}(n-1,1)$.

The phases are therefore obtained by ***recursion****. Other methods exist, also.*

DIP&E-17.6 Bispectrum in Speckle Restoration

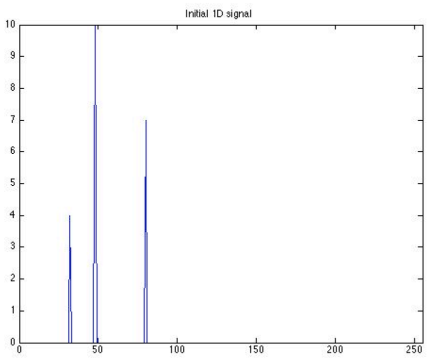
An estimate of the SNR of each phase value, obtained from the bispectrum and the noise on previous phase estimates, is used to form a weighted average of the phase at each point in the reconstruction.

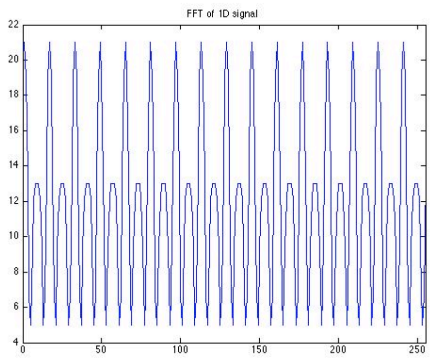
When noise is present, it can lead to 2π mismatches in the multiple phase estimates for each frequency (see Northcott, Ayers, and Dainty for details).

- *Use traditional speckle techniques to estimate the modulus of the object.*
- *Decide which bispectrum values are to be calculated.*
- *Calculate the Fourier transform of each frame. From this calculate the mean value of each of the chosen bispectrum points, over the many speckle data set.*
- *Using the bispectrum points, reconstruct an estimate of the object transform phases.*

DIP&E-17.7 Bispectrum in Speckle Restoration

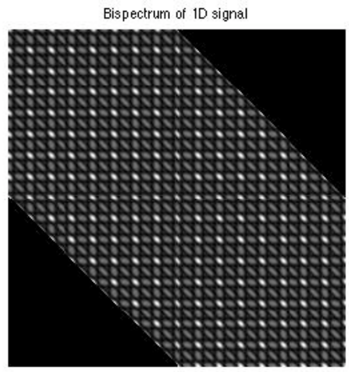
Let us form the bispectrum of an example 1D signal, from its 1D FFT:

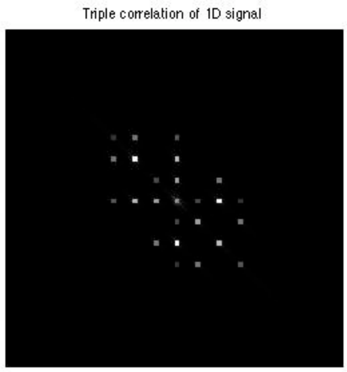




DIP&E-17.8 Bispectrum in Speckle Restoration

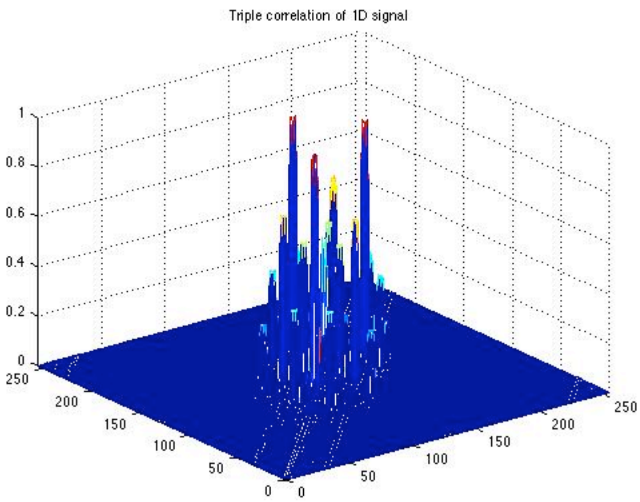
Here are the resulting 2D bispectrum, and triple correlation (by inverse 2D FFT)





DIP&E-17.9 Bispectrum in Speckle Restoration

The triple correlation in 3D graphics:



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The MATLAB program that created the previous examples.

Note how easily the 2D bispectrum is computed, after forming the 1D fft, by a double loop in u and v (i.e., u_1 and u_2).

```
% bispect1D.m
%
% computes the bispectrum (2D) of a 1D signal, im, displays it, then
%  computes the inverse FFT of the 2D bispectrum,
%  which is therefore the 2D triple correlation of the 1D signal
%
% Msy 2008

close all
clear all

n = 256;
nhlf = n/2;
nhlfp = nhlf+1;
nhlfm = nhlf-1;

nbi = n;
nbihlfp = nbi/2 +1;

im = zeros(1, n);  % set up the 1D signal - zeros with some impulses

isw = input('Enter one of two built-in signals (0 or 1): ');

if isw == 0
    im(8) = 4;
    im(12) = 10;
    im(20) = 7;
else
    im(32) = 4;
    im(48) = 10;
    im(80) = 7;
end

figure(1)  % display the 1D signal
plot(im)
title('Initial 1D signal')
set(gca,'XLimMode','manual','XLim',[0 n-1])
pause(3)

fim = fft(im);  % FFT the 1D signal and display
sfim = fftshift(fim);
figure(2)
plot(abs(sfim))
title('FFT of 1D signal')
set(gca,'XLimMode','manual','XLim',[0 n-1])
pause(3)

bispect = zeros(nbi, nbi);  % compute the bispectrum of the 1D signal
for v = -nhlf:nhlfm
    for u = -nhlf:nhlfm
        if abs(u+v) <= nhlfm
            bispect(nbihlfp +v, nbihlfp +u) = sfim(nhlfp +u).*sfim(nhlfp +v).*conj(sfim(nhlfp +u+v));
        end
    end
end
```

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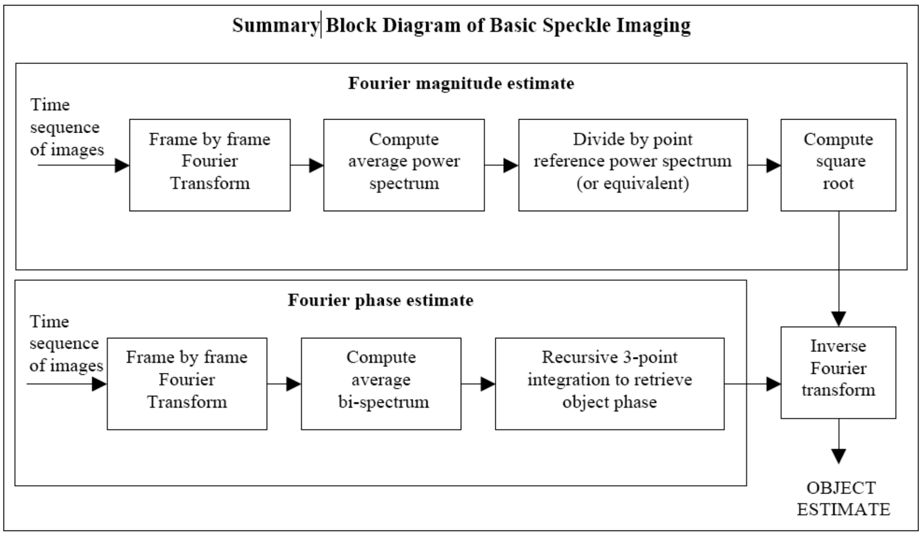
The second half of the bispectrum program.

(note that the bispectrum computation loops are shown again ou of interest.)

```
bispect = zeros(nbi, nbi);  % compute the bispectrum of the 1D signal
for v = -nhlf:nhlfm
    for u = -nhlf:nhlfm
        if abs(u+v) <= nhlfm
            bispect(nbihlfp +v, nbihlfp +u) = sfim(nhlfp +u).*sfim(nhlfp +v).*conj(sfim(nhlfp +u+v));
        end
    end
end

bispectud = flipud(bispect); % display the 2d bispectrum
figure(3)

scale = max(max(bispectud));
bispectud = bispectud/scale;
imshow(abs(bispectud))
title('Bispectrum of 1D signal')
pause(3)

bispectorig = fftshift(bispect);

tricorr = ifft2(bispectorig);  % compute the triple correlation (inverse FFT of bispectrum)

figure(4)
stricorr = fftshift(tricorr);
stricorr(nbihlfp, nbihlfp) = 0; % zero out centre term (usually too large, swamps display)

nbig = 4;  % expand the triple correlation for display
temp = zeros(nbi+nbig-1, nbi+nbig-1);
for yind = 1:nbig
    for xind = 1:nbig
        temp(yind:yind+nbi-1, xind:xind+nbi-1) = temp(yind:yind+nbi-1, xind:xind+nbi-1) + stricorr;
    end
end

imdisp = temp(1:nbi, 1:nbi); % scale and display triple correlation (grey scale)
imdisp = abs(imdisp);
maxval = max(max(imdisp));
minval = min(min(imdisp));
scale = maxval - minval;
imdisp = (imdisp-minval)/scale;

imshow(imdisp)
title('Triple correlation of 1D signal')
pause(3)

x = [0:nbi-1];
y = x;

figure(5)
mesh(x,y,imdisp) % display triple correlation as 3D surface
title('Triple correlation of 1D signal')
set(gca,'XLimMode','manual','XLim',[0 n-1])
set(gca,'YLimMode','manual','YLim',[0 n-1])
```

DIP&E-17.12 Bispectrum in Speckle Restoration

Typical method using bispectrum in speckle imaging (Carrano 2002, fig. 2)



DIP&E-17.13 Bispectrum in Speckle Restoration

Example, bispectrum reconstruction of water reflections (Zhiying Wen, 2007)