SPECKLE IMAGING WITH THE BISPECTRUM AND WITHOUT REFERENCE STAR

E. THIEBAUT
Observatoire de Meudon – FRANCE

January 20, 1993

Abstract. A technique for reconstructing diffraction-limited image of an object from speckle images without reference star is applied to both simulated and real data. The object spectrum is estimated by blind deconvolution using the power spectrum of the speckle images and the phase is restored from the bispectrum.

Key words: Bispectrum – Image Reconstruction – Blind Deconvolution

1. Presentation Of The Method

1.1. Image Formation

The observed image $i(x)$ is the convolution (noted $*$) of the object intensity distribution $o(x)$ by the point spread function $s(x)$ of the combined telescope and atmosphere (Roddier, 1981):

$$i(x) = s(x) * o(x). \tag{1}$$

Fourier transformation leads to:

$$I(u) = S(u)O(u). \tag{2}$$

When recorded with an exposure time much shorter than the evolution time of atmospheric phase perturbations, the highly magnified observed image shows a pattern of speckles. It has been established for a long time (Labeyrie, 1970) that such speckled images contain diffraction-limited information on the spatial structure of the observed object.

1.2. The Power Spectrum

The mean power spectrum of speckle images can be written as (Dainty et al., 1978):

$$\left\langle |I(u)|^2 \right\rangle = \left\langle |S(u)|^2 \right\rangle |O(u)|^2 \simeq |\langle I(u)\rangle|^2 + N_{\rm S} T(u)\, |O(u)|^2\,, \tag{3}$$

where $N_{\rm S} \simeq 2.30 D/r_0$ is the mean number of speckles per image and $T(u)$ the normalized transfer function of the telescope.

1.3. The Bispectrum

The bispectrum of the speckle images is:

$$\left\langle I^{(3)}(u,v) \right\rangle = \langle I(u) I(v) I^*(u+v) \rangle = \left\langle S^{(3)}(u,v) \right\rangle O^{(3)}(u,v). \tag{4}$$

The phase $\beta(u,v)$ of $\left\langle I^{(3)}(u,v)\right\rangle$ tends to be unaffected by atmospheric perturbations and may be uncorrupted by telescope aberrations (Roddier, 1988). Hence $\beta(u,v)$ can be approximated by the object bispectrum phase, thus:

$$e^{\jmath\beta(u,v)} \simeq e^{\jmath\theta(u)} e^{\jmath\theta(v)} e^{-\jmath\theta(u+v)}, \tag{5}$$

where $\theta(u)$ is the phase of the object spectrum.

1.4. Phase Reconstruction

The bispectrum subplanes $\left\langle I^{(3)}(u,v_0)\right\rangle$, with $||v_0|| \leq r_0/\lambda$, are used to reconstruct a weighted-least-squares estimation of the object spectrum phasor $e^{\jmath\theta(u)}$. This is done by an iterative method based on the one described by Matson (1991). Basically, the new estimation $e^{\jmath\theta_{k+1}(u)}$ for $e^{\jmath\theta(u)}$ is given by:

$$e^{\jmath\theta_{k+1}(u)} \propto \sum_{v_0} \frac{e^{\jmath\beta(u,v_0)} e^{-\jmath\theta_k(v_0)} e^{\jmath\theta_k(u+v_0)}}{\sigma_\beta^2(u,v_0)}, \tag{6}$$

where the previously estimated phasor $e^{\jmath\theta_k(u)}$ is set to zero if not already known so as to cancel its contribution to the weighted sum. The expression of the bispectrum phase variance $\sigma_\beta^2(u,v)$ is given in Ayers (1988).

1.5. Image reconstruction

The power spectrum $\left\langle |I(u)|^2\right\rangle$ combined with the phasor $e^{\jmath\theta(u)}$ given by the bispectrum can be written as the convolution of the estimated object $\widetilde{O}(u)$ by some transfer function $H(u)$:

$$\sqrt{\left\langle |I(u)|^2\right\rangle} e^{\jmath\theta(u)} \simeq H(u)\widetilde{O}(u). \tag{7}$$

Following Lane (1992), this complex spectrum is blindly deconvolved with positivity (and possibly support) constraints for $\widetilde{o}(x)$ and $h(x)$. A further constraint of symmetry is used for $h(x)$ (i.e. $H(u)$ real). Moreover, Eq. 3 leads to a (possible) first estimation of the smoothed object power spectrum:

$$\left|\widetilde{O_0}(u)\right|^2 = \left\langle |I(u)|^2\right\rangle - |\langle I(u)\rangle|^2. \tag{8}$$

Simulation of 100 speckle images (free from Poisson noise) of a complex triple object were used to test the reconstruction process. Effects of turbulence were simulated with $D/r_0 \simeq 10$, and 19 subplanes were used to reconstruct the phase of the object. Using conjugate gradient minimization, the final estimation of the object is obtained after ~ 100 iterations. This final estimation nicely fits the smoothed original object.

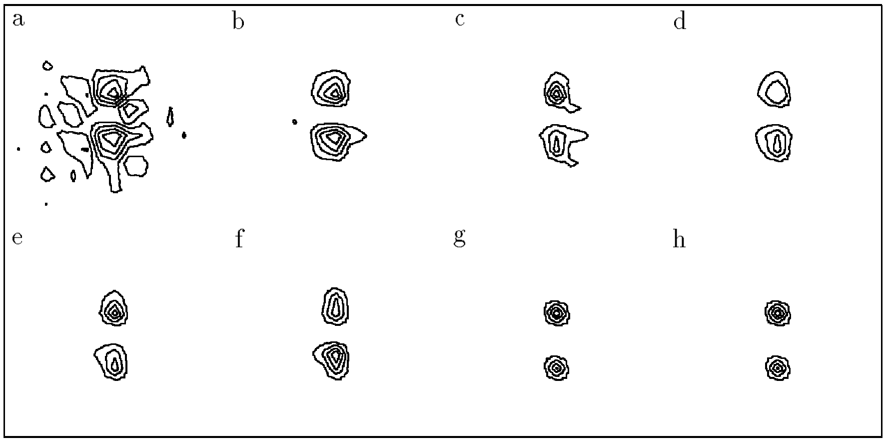

Fig. 1. Improvement of the image of Capella by applying the constraint of positivity with an increasing number of iterations: (a) starting estimation, (b) 1st iteration, (c) 5th iteration, (d) 10th iteration, (e) 20th iteration, (f) 30th iteration, (g) 40th iteration, (h) 50th iteration.

2. Results: Capella

A set of 200 speckle images of Capella (αAurigae) were used to reconstruct an image of this object (data courtesy of ONERA – FRANCE). These observations were performed at the 4.2 m William Herschel Telescope on La Palma during the night November 8, 1990 for the purpose of wave front sensor *a posteriori* deconvolution (Michau et al., 1991). Capella is a bright binary star ($m_V \simeq 0.08$, separation $\simeq$ 55 marcsec at the acquisition date). The results shown in Fig. 1 are obtained from 19 bispectrum subplanes, they are at least as good as deconvolution with wavefront sensor (Michau et al., 1991).

References

G. R. Ayers, 1988, *Correlation techniques for imaging through atmospheric turbulence*, thesis, Imperial College of Science, Technology and Medicine, University of London, London

J. C. Dainty and A. H. Greenaway, 1978, *Estimation of spatial power spectra in speckle interferometry*, J. Opt. Soc. Am., Vol. 69, No. 5, pp. 786-790

A. Labeyrie, 1970, *Attainment of diffraction-limited resolution in large telescopes by Fourier analyzing speckle patterns in star images*, Astron. Astrophys., Vol. 6, pp. 85-87

R. G. Lane, 1992, *Blind deconvolution of speckle images*, J. Opt. Soc. Am. A, Vol. 9, No. 9, pp. 1508-1514

C. L. Matson, 1991, *Weighted-least-squares phase reconstruction from the bispectrum*, J. Opt. Soc. Am. A, Vol. 8, No. 12, pp. 1905-1913

V. Michau, T. Marais, J. Laurent, J. Primot, J.-C. Fontanella, M. Tallon, J. Fuensalida, 1991, *High-resolution astronomical observations using deconvolution from wavefront sensing*, SPIE's International Symposium on Optical Engineering and Photonics in AerospaceSensing, Orlando, FL (USA)

F. Roddier, 1981, in *Progress in Optics*, ed. E. Wolf (North-Holland Publishing Company, Amsterdam) Vol. 19, pp. 281-376

F. Roddier, 1988, *Interferometric imaging in optical astronomy*, Physics Report